OPTIMIZATION AND PORTFOLIO CONSTRUCTION

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INTRODUCTION

In an earlier paper[1] we asked the question "Is it possible that the underperformance of so many mutual fund managers versus their benchmarks is due to their portfolio construction skills and not their stock selection skills?" We, like Day, Wang and Xu[2], and others answer this question with a rousing "yes" – many of the stock and bond fund managers we, and others, have looked at do a good job of selecting securities, but not as good a job when it comes to assembling these securities into a portfolio.

We found that using two common techniques – the first a CAPM-based construction method written about by Bill Sharpe more than 30 years ago[3] and the aforementioned work of Day, Wang and Xu, which is based on the justly famous Fama-French multi-factor models – can substantially improve a portfolio's return. As a matter of fact, in a number of cases, the multi-factor method adds enough basis points per month that many funds power past their benchmark on a consistent year-over-year basis. Further, we found[4] that a new technique at the time, G-CAPM, could also be a powerful tool in portfolio construction.

[1] A. Clark, "Securities Selection and Portfolio Optimization: Is Money Being Left on the Table?" *Lipper FundIndustry Insight Report*, June 22, 2005. Also available, with M. Labovitz as co-author, in the *Proceedings of the Financial Engineering and Applications Conference 2006*, ACTA Press.
[2] T. Day, Y. Wang and Y. Xu, "Investigating Underperformance of Mutual Fund Portfolios," http://www.utdallas.edu/%7Eyexiaoxu/Mfd.PDF
[3] W.F. Sharpe, "Imputing Expected Returns From Portfolio Composition," Journal of Financial and Quantitative Analysis, June 1974
[4] A. Clark, *op.cit.*

Subsequent to our initial work, we looked at other means of using optimization to improve portfolio performance. This paper will focus on two techniques: The use of random matrix theory (RMT[5]) in portfolio construction, and dynamic programming.

RMT

The statistical properties of the correlations between returns of different stocks have been investigated in both economics and physics, but with different goals. In economic research, a main goal is to determine the number of *k* factors present in the market using arbitrage pricing theory (APT). In this theory, an economic factor is a factor that is common to a set of stocks, i.e., *n* one period asset returns R_n are generated by a linear process with *k* factors.

$$R_n = R_{n0} + B\xi_k + \varepsilon_n$$

Where R_{n0} represents the risk-free and factor-risk premia mean returns, *B* the *n x k* matrix of factor weights, ξ_k the times series of *k* factors affecting the asset returns, and ε_n an asset specific risk. This APT equation is the general formulation of such widely known models as Fama-French and the Barra risk models, though, from an economic standpoint, there are interpretational differences between the various models.

[5] The mathematics behind RMT will not be discussed in this paper, just the results of its use. Interested readers can look at J-P Bouchard and M. Potters' **Theory of Financial Risks**, Cambridge University Press, 2001, or H. Stanley and M. Mantegna's **Introduction to Econophysics**, Cambridge University Press, 2000 for a good introduction to RMT.

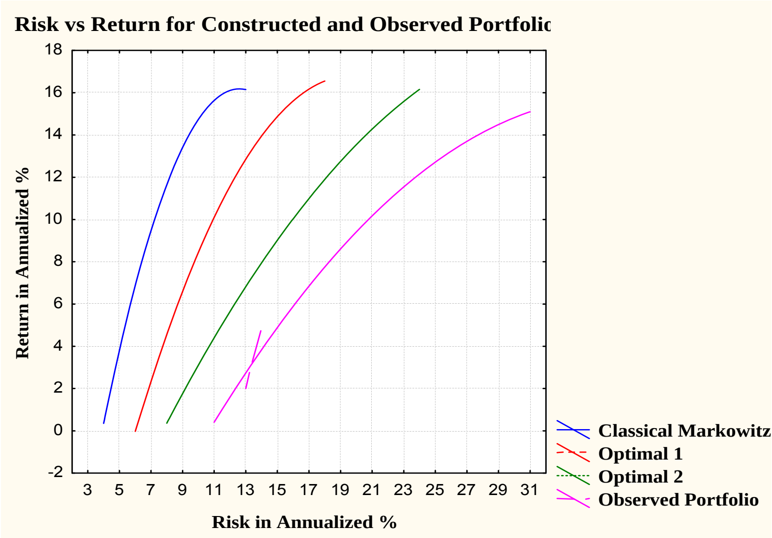
Within the framework of APT, the existence of eigenvalues dominating the correlation matrix has been interpreted as evidence of a small number of *k* factors driving the dynamics of asset returns. Empirical analysis by economists seems to suggest that only a few *k* factors exist, and that there is strong evidence of a prominent *k* factor among them (similar to, if not the same as, the market factor in CAPM).

When physicists have examined the same correlation of returns using RMT, they too have noticed multiple factors influencing stock returns. They have also noted that not "cleaning" the correlation matrix prior to looking for factors seriously compromises any use of the correlation or covariance matrix, whether that it is being used for factor study, risk assessment or portfolio construction.

The reason for this is that unless the number of periods observed is much larger than the number of stocks or factors being looked at, the correlations in such a matrix will be dominated by noise. According to papers by Kondor and Pafka[6], measurement noise is a serious issue in calculating correlation of covariance matrices if the ratio of number of securities divided by the number of observations is 0.6 or greater. Only if that ratio gets to 0.2 or smaller do Kondor and Pafka not recommend cleaning the matrix. The author of this paper recommends that when the ratio is 0.2 or smaller, "subtracting" the market factor from the time series' of asset returns before computing correlation or covariance matrices helps brings out the "true" relationship between stocks, which is very helpful when doing portfolio optimization. This subtraction can be done by taking an index such at the S&P 500, regressing the stock returns on it and using the regression residuals as the input for constructing the covariance or correlation matrix.

The text cited above, by Bouchard and Potters, has a very revealing graph (reproduced below) which shows clearly the impact of cleaning correlation matrices:

Figure 1



In Figure 1, using the empirical correlation matrix (Classical Markowitz in blue) leads to a dramatic underestimation (by a factor of 3!) of the real risk (in blue) by over-investing in artificially low-risk stocks (eigenvectors). The risk obtained using either of the "cleaned" portfolios (Optimal 1 in red and 2 in green) is more reliable (the realized risk is now only a factor 1.5 larger than the predicted risk), although the real risk (Observed Portfolio in pink) is always larger than the predicted one. This comes from the fact that any amount of uncertainty in the correlation matrix produces, via the optimization procedure, a bias towards low risk portfolios. Readers interested in the technical details of "cleaning" matrices and then optimizing portfolios are referred to papers by Rosenow *et al*[7] and others such those of Bouchard and Potters and Kondor and Pafka.

[6] I. Kondor and S. Pafka "Noisy Covariance Matrices and Portfolio Optimization," European Physical Journal B, Vol. 27, 2002, and "Noisy Covariance Matrices and Portfolio Optimization II," Physica A, 2003

[7] B Rosenow, V. Plerou, P. Gopikrishnan, and H.E. Stanley, "Portfolio Optimization and the Random Magnet Problem," http://arxiv.org/PS_cache/cond-mat/pdf/0111/0111537v1.pdf

The author has found other uses for RMT and has written about them in other papers[8] and he has discussed with H. Turowski the use of ultrametrics, a tool similar to RMT, to determine the strength of the linkage or correlations between stocks in similar sectors or industries. Turowski and Clark both found that the existence of the link (or its lack thereof) and its strength (correlation coefficient) varies over time. Turowski and Clark also found that in many cases, having exposure to just one stock (the "mother stock" if it exists) is sufficient if one is seeking to gain exposure to a particular sector or industry. The implications here are that RMT is a powerful tool that needs to be used more in the financial and economic communities.

Dynamic Programming

The term "dynamic programming" was originally used in the 1940's by Richard Bellman to describe the process of solving problems where one needs to find the best decisions one after another. Bellman's contribution to the subject is remembered in the name of the Bellman equation, a central result of dynamic programming, which restates an optimization problem in recursive form.

The word "programming" in "dynamic programming" has no particular connection to computer programming at all, and instead comes from the term "mathematical programming", a synonym for optimization. Thus, the "program" is the optimal plan for action that is produced. For instance, a finalized schedule of events at an exhibition is sometimes called a program. Programming, in this sense, means finding an acceptable plan of action, an algorithm.

The acceptable plan of action Thomson Reuters' researchers Mark Labovitz and Tod Morrison found earlier this year for target-maturity funds has had a major impact on the construction, performance assessment, and improved understanding of same.[9]

Their work on glide paths – the mix of assets and asset types that target-maturity funds used to generate returns – has shown that treating the glide path as an optimization over time has very beneficial results. In the words of the paper:

> To this end we solve within a complex but single model an optimization for maximizing wealth. We solve it recursively for every time step. In order to give the glide path a quasi-personalized feel, we build a target trajectory of one of a set of chosen metrics into the constraint set (at this time we are using risk-aversion). Consequently, for each time step the model suggests an adjustment to the existing trajectory to move the intermediate step back toward the desired trajectory. This step-wise-adjusting nature of the model yields a "dynamic optimization"; variants of it are commonly used in physical models such as controlling rockets as well as in models for controlling more intangible quantities such as wealth.

No glide path ever follows a smooth, ever-growing trajectory, and like the rocket mentioned in the paper quoted above, mid-course corrections are needed *and* expected so the rocket – or, in our case, your retirement money – reaches its goal: Either the moon or the dollar amount you decided you need to retire comfortably.

As mentioned in the quote above, in the Labovitz-Morrison model, the investigators incorporate a risk-aversion model to form a target trajectory. By risk aversion, the authors mean the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain with a more certain, but possibly lower, expected payoff or return. As an FYI, the inverse of a person's risk aversion is sometimes called his/her risk tolerance.

In Labovitz and Morrison's research, they used two risk-aversion models, one risk-neutral and

[8] A. Clark, "The U.S. Dollar: Which Funds to be in When it Climbs and Which Funds to be in When it Falls," *Lipper FundIndustry Insight Report*, November 19, 2004 and "Small-Cap Funds Versus Large-Cap Funds: Drivers of Performance and When to Reallocate," *Lipper FundIndustry Insight Report*, September 2004.

[9] M.L. Labovitz and T. Morrison, "Target Maturity Funds Research at Lipper with an Emphasis on Dynamic Optimization of Glide Paths," *Lipper Research Series*, April 21, 2008

one risk-averse. While these are, laughingly, not exhaustive, as noted in their paper, their point is, that by segmenting clients or prospects, into one of four or five risk-aversion models, they were able to give the glide path a quasi-personalized feel. This is an important step forward in the construction of target-maturity funds. No funds at this point in time publically state what their risk aversion (or risk tolerance) is. Investors and financial advisors have no way of knowing if the target-maturity fund or funds they are looking at fit their risk profile. Without being able to do this, financial advisors (and plan sponsors) have difficulty fulfilling their fiduciary responsibility (part of which is to assess the appropriateness of an investment for a client), which has clear ramifications for the investing public, which is taking to target-maturity funds in droves.

The Labovitz-Morrison model can also address such issues as: 1) Shortfall probability studies for existing glide paths, i.e., how likely are you to reach your retirement goal *and* what steps can be taken if your current plan cannot get you there, 2) Calculate formal rankings of target maturity fund firms within a large variety of cohort groupings, 3) Suggest prototype glide paths to target-maturity providers under various client-defined sets of constraints and 4) Provide benchmarking for firms within their classification or against a custom peer group. This is just a small list of how Thomson Reuters plans to use the Labovitz-Morrison model.

Conclusion

This has been a brief summary of the portfolio construction and optimization work Thomson Reuters' employees have been working on for the past two years. RMT is already institutionalized in our processes as it is part of what lies behind the success of Optimal Indices. Soon we will begin to gear-up work on target-maturity funds and other retirement plan vehicles we think would benefit from the Labovitz-Morrison approach outlined above.

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